Constraints from the old quasar APM 08279+5255 on two classes of $\Lambda(t)$-cosmologies

J. F. Jesus*
Instituto de Astronomia, Geofísica e Ciências Atmosféricas, USP, Rua do Matão, 1226, São Paulo, SP, Brasil

February 5, 2008

Abstract

The viability of two different classes of $\Lambda(t)$CDM cosmologies is tested by using the APM 08279+5255, an old quasar at redshift $z = 3.91$. In the first class of models, the cosmological term scales as $\Lambda(t) \sim R^{-n}$. The particular case $n = 0$ describes the standard ΛCDM model whereas $n = 2$ stands for the Chen and Wu model. For an estimated age of 2 Gyr, it is found that the power index has a lower limit $n > 0.21$, whereas for 3 Gyr the limit is $n > 0.6$. Since n can not be so large as ~ 0.81, the ΛCDM and Chen and Wu models are also ruled out by this analysis. The second class of models is the one recently proposed by Wang and Meng which describes several $\Lambda(t)$CDM cosmologies discussed in the literature. By assuming that the true age is 2 Gyr it is found that the ϵ parameter satisfies the lower bound $\epsilon > 0.11$, while for 3 Gyr, a lower limit of $\epsilon > 0.52$ is obtained. Such limits are slightly modified when the baryonic component is included.

1 Introduction

Many cosmological models driven by dark energy have been proposed in the literature for explaining the discovered cosmic acceleration of the Universe [1]. At present, the preferred scenario is provided by the so-called ΛCDM model [2]. However, this model is plagued with the Cosmological Constant Problem (CCP) [3]. Such a problem occurs because the value of the Cosmological Constant (CC) suggested by the recent observations is incredibly small as compared to simple estimates from Quantum Theory of Fields (about 120 orders of magnitude smaller).

Decaying Vacuum Cosmologies or Λ(t)-Cosmologies [4] is a possibility to alleviate this problem. This kind of models try to explain the CCP through the hypothesis that Λ couples with matter in such a way that it decays with time. Qualitatively, this means that the value of Λ is very small today because the Universe is too old.

Λ(t)-cosmologies are defined by some direct (or indirect) phenomenological time dependence of Λ whose free parameters must be constrained by the cosmological tests. One of these tests is the age of the universe at high redshifts

*E-mail: jfernando@astro.iag.usp.br

[5]. Since the Universe must be older than any object contained in it, the basic idea of this test is to compare the age of the Universe at a given redshift with the estimated age of the oldest objects at the same redshift.

In this paper, the age test is applied for two classes of Λ(t)-cosmologies. The first one generalizes the Chen and Wu model [6] by considering a cosmological term scaling as $\Lambda(t) \sim R^{-n}$. The second class is the one recently proposed by Wang and Meng [7]. In this scenario, the energy density of the CDM component scales as $\rho_m \sim R^{-3+\epsilon}$. The prediction of both models are confronted to the old quasar APM 08279+5255 at redshift z = 3.91, whose age was recently estimated trough its iron abundance [8, 9].

2 Extended Chen and Wu Model

In 1990, by using dimensional arguments, Chen and Wu [6] proposed the following functional form for the cosmological term:

$$\Lambda = \frac{\alpha}{R^2}, \tag{1}$$

where α is a dimensionless constant and $R(t)$ is the scale factor of the FRW type geometries. The power $n = 2$ was fixed by considering that gravity is described by a classical field after the Planck age. In what follows, by assuming that the vacuum energy density, and the cosmic acceleration at the present stage may have a quantum origin, we discuss the general $\Lambda(t)$ dependence [10]

$$\Lambda = \frac{\alpha}{R^n}. \tag{2}$$

An advantage of this functional form is that the Chen and Wu model is recovered for $n = 2$ whereas, for $n = 0$, it reduces to the cosmic concordance (ΛCDM) model.

Now, by assuming spatial flatness as predicted by inflation and observationally suggested by the WMAP experiments [11], the FRW equation plus the energy conservation law can be written as

$$8\pi G\rho_m + \frac{\alpha}{R^n} = 3H^2, \tag{3}$$

$$\frac{d}{dR}\left(\rho_m R^3\right) = \frac{n\alpha}{8\pi G R^{n-2}}, \tag{4}$$

where ρ_m is the energy density for the pressureless cold dark matter (CDM) fluid, and $H = \dot{R}/R$ is the Hubble parameter.

Now, let us determine the expression for t_z, the age of the Universe at redshift z. First, one must integrate the energy conservation law for obtaining the ρ_m, and inserting the result into the FRW equation, it thus follow that

$$H^2 = H_0^2\left[\left(\frac{n-3\Omega_{mo}}{n-3}\right)\left(\frac{R_o}{R}\right)^3 + \frac{3\Omega_{\Lambda 0}}{3-n}\left(\frac{R_o}{R}\right)^n\right]. \tag{5}$$

Note that for $n = 0$ this expression for the Hubble parameter reduces to the one of the flat ΛCDM model, as should be expected. Finally, by integrating again, we have for t_z:

$$t_z = H_0^{-1} \int_0^{(1+z)^{-1}} \left[\left(\frac{n - 3\Omega_{mo}}{n-3} \right) x^{-1} + \frac{3(1-\Omega_{mo}) x^{2-n}}{(3-n)} \right]^{-\frac{1}{2}} dx, \qquad (6)$$

where $x = R/R_o$ is a convenient integration variable. It is straightforward to find similar expressions for the logarithm solution ($n = 3$), but this case will not be considered here. It can also be shown that n can not be so large as one wishes. Actually, the weak energy condition for the matter density implies an upper bound $n \sim 0.81$ (see Appendix).

3 The Wang and Meng (WM) Model

More recently, a new decaying vacuum cosmology was proposed by Wang and Meng [7]. Unlike the common approach in the literature, they did not assume a functional form for Λ(t). In such a scenario, the decay law is deduced from its effect on the CDM evolution. Qualitatively, since the vacuum is decaying into CDM particles, the created matter must dilute more slowly in comparison with the standard evolution ($\rho_m \propto R^{-3}$). Thus, assuming that the deviation from the conservation law is characterized by a positive constant ϵ, one may write

$$\rho_m = \rho_{mo} \left(\frac{R}{R_o} \right)^{-3+\epsilon}, \qquad (7)$$

where ϵ is a small positive constant and ρ_{mo} is the present value of ρ_m. Inserting the above expression in the energy conservation law, one finds

$$\left(\frac{R_o}{R} \right)^3 \frac{d}{dR} \left(\rho_{mo} \left(\frac{R}{R_o} \right)^{\epsilon} \right) = -\frac{d\rho_\Lambda}{dR}. \qquad (8)$$

Hence, by integrating the above expression, it thus follows that

$$\rho_\Lambda = \tilde{\rho}_{\Lambda 0} + \frac{\epsilon \rho_{mo}}{3-\epsilon} \left(\frac{R}{R_o} \right)^{-3+\epsilon}, \qquad (9)$$

where $\tilde{\rho}_{\Lambda 0}$ is an integration constant which can be associated to "the ground state of vacuum" (note that the present vacuum energy density is $\rho_{\Lambda 0} = \tilde{\rho}_{\Lambda 0} + \epsilon \rho_{mo}/(3-\epsilon)$).

Under such conditions, the FRW equation for the WM model reads:

$$H^2 = \frac{8\pi G}{3} (\rho_m + \rho_\Lambda) = H_0^2 \left[\frac{3\Omega_{mo}}{3-\epsilon} \left(\frac{R_o}{R} \right)^{3-\epsilon} + \tilde{\Omega}_{\Lambda 0} \right], \qquad (10)$$

with an age-redshift relation given by

$$t_z = H_0^{-1} \int_0^{(1+z)^{-1}} \frac{dx}{\sqrt{(1-\tilde{\Omega}_{\Lambda 0}) x^{\epsilon-1} + \tilde{\Omega}_{\Lambda 0} x^2}}. \qquad (11)$$

As should be expected, by taking $\epsilon = 0$ in the above expression (no CDM creation), the age-redshift relation for the ΛCDM model is recovered.

4 The Age-Redshift Test

Let us now discuss some constraints by considering the quasar APM 08279+5255, as a cosmic clock. Such a quasar, located at $z = 3.91$, has an estimated age from 2 to 3 Gyr, with a best fit age of 2.1 Gyr [8, 9]. As first discussed by Alcaniz and Lima [5], the age test here is defined by the condition

$$t_z \geq t_g, \tag{12}$$

where t_z is the age of the Universe at redshift z and t_g is the age of the quasar at the same redshift. It just says that the Universe is older than any structure that it contains.

For test both models we have used the optimal value of the matter density parameter as given by the WMAP team [11], $\Omega_{mo} = 0.27 \pm 0.04$, whereas the Hubble parameter is the one provided by the HST team [12], $H_0 = 72 \pm 8 kms^{-1}Mpc^{-1}$. It is convenient to introduce a dimensionless age parameter, $T_g = H_0 t_g$, in such a way that for the 2.0 Gyr old quasar, this value of the Hubble parameter constrains it to the range $0.131 \leq T_g \leq 0.163$. It follows that $T_g \geq 0.131$. Therefore, for a given value of H_0, only models with an expanding age bigger than this value at $z = 3.91$ will be compatible with the existence of this object. In order to assure the robustness of our analysis, it is also necessary to adopt the lower bound for the value of the Hubble parameter, $H_0 = 64 kms^{-1}Mpc^{-1}$.

4.1 Constraints on Chen and Wu Model

The results for the Chen and Wu model can be seen on Table 4.1. The constraints on the power n are heavily dependent on the estimated age of the quasar. By considering that the correct age is 2 Gyr, the power n has a lower limit given by $n \geq 0.21$. This minimal value increases for 0.28 when we consider the optimal estimated age of 2.1 Gyr and it is $n > 0.6$ if the age is 3.0 Gyr. In particular, these results imply that the ΛCDM ($n = 0$) is ruled out by this analysis, if we have $\Omega_{\Lambda 0} = 0.73$. This result agrees with the ones recently determined by some authors for the standard ΛCDM model using the same quasar. They found that $\Omega_{\Lambda 0} \geq 0.78$ in order to be compatible with the existence of this quasar [13].

Table 1. Values of n_{min} for the three estimated ages of quasar

t_g (Gyr)	n_{min}
2	0.21
2.1	0.28
3	0.60

4.2 Constraints on Wang and Meng Model

As in the previous model, we have made use of a routine in C to perform the age test for the WM model. The main results of our analysis are displayed in Table 4.2. As can be seen there, the minimal value of the free parameter ϵ is

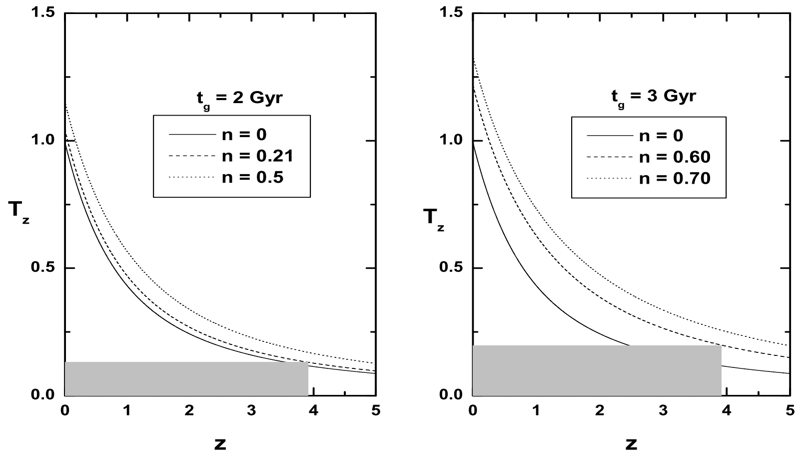

Figure 1: t_z *versus* redshift for two estimated ages of the quasar. Left panel: 2 Gyr. Right panel: 3 Gyr. The curves correspond to each value of n as indicated. The rectangle corresponds to the age of the quasar.

modified when the baryonic component is included. This kind of model was recently considered by Alcaniz and Lima [14]. Note that the minimum value of ϵ varies in the range $0.115 < \epsilon < 0.527$.

Table 2. Values of ϵ_{min} for the three estimated ages of quasar (with and without baryons)

t_g (Gyr)	ϵ_{min} (no baryons)	ϵ_{min} (with baryons)
2	0.115	0.231
2.1	0.163	0.296
3	0.527	0.910

It is also interesting to know how the minimal value of ϵ depends on matter density parameter. In Figure 2, the values of ϵ_{min} are plotted as a function of Ω_{mo} for the WM model with no baryons. We see that ϵ_{min} is heavily dependent on the values of Ω_{mo}. It becomes negative for $\Omega_{mo} < 0.2$ whether the age of the quasar is 2 Gyr. The lower limit is nearly 0.1 for 3 Gyr. It is worth notice that negative values of ϵ are thermodynamically forbidden [14].

Alcaniz and Lima [14] also analyzed the WM model using Sne Ia, X-Ray luminosity from galaxy clusters and CMB data. A best fit of $\epsilon = 0.06 \pm 0.10$ at 95% c.l. was established in their joint analysis. Concerning the present age test, this result is not compatible with the existence of the quasar APM 08279+5255 (see Table 2). Actually, even for an estimated age of 2.1 Gyr, the best fit found on Ref. [9], the above value disagrees from our minimal value by 3 standard deviations. Naturally, if the real age is 3 Gyr the situation becomes worst. Although improving the determination of the critical redshift, the presence of a baryonic component worsens the scenario for the age test because it decelerates the Universe thereby decreasing the age at a given redshift.

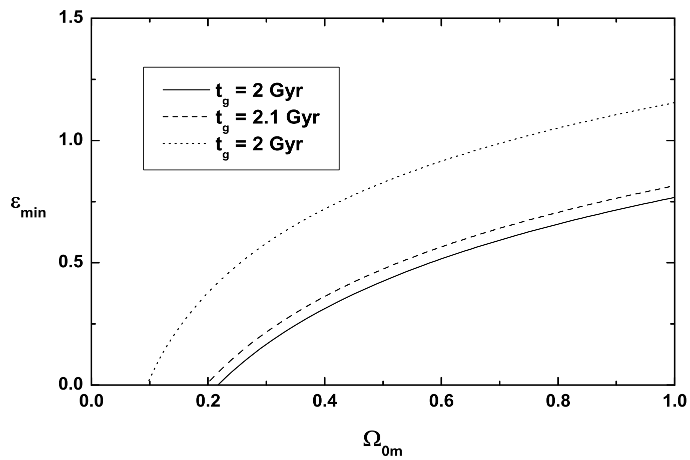

Figure 2: The parameter ϵ_{min} *versus* Ω_{mo} for the WM model with no baryons. As indicated in the panel, 3 different estimated ages of the quasar has been considered.

5 Conclusions

The age test has been applied for two different $\Lambda(t)$ cosmologies by using the old quasar APM 08279+5255. For $\Lambda \sim R^{-n}$ (extended Chen and Wu model) we found $n \geq 0.21$. This means that the cosmic concordance ΛCDM model ($n = 0$) is incompatible with the existence of this object. This result is in line with the previous analysis by Alcaniz et al. [13]. However, there is an upper limit on n from physical and observational considerations. In particular, the original CW model ($n = 2$) is not compatible with the WMAP data because it requires $\Omega_{mo} > 2/3$ (see also Appendix).

Finally, we have established new limits to the ϵ parameter of the Wang and Meng model (with and with no baryons) which are more stringent than the ones recently determined by Alcaniz and Lima [14]. Until the present, the existence of the old quasar constrains severely all the models present in the literature.

Acknowledgments

This work was supported by CNPq (Brazilian Research Agency). I would like to thank Prof. J. Ademir S. Lima for suggesting the problem. I am also grateful to Rodrigo Holanda and Rose Santos for the encouragement and helpful discussions.

A Extended CW model and its maximum power n

In the extended CW model, the matter density reads (see Eqs. (3)-(5)):

$$\rho_m = \frac{3H_0^2}{8\pi G}\left[\frac{n-3\Omega_{mo}}{n-3}\left(\frac{R_o}{R}\right)^3 + \frac{n\Omega_{\Lambda 0}}{(3-n)}\left(\frac{R_o}{R}\right)^n\right]. \tag{13}$$

Now, supposing $n < 3$, it is easy to see that the matter density is always positive only if $n \leq 3\Omega_{mo}$. Thus, n should be less than ~ 0.81 for $\Omega_{mo} = 0.27$ as suggested from CMB experiments [11]. Actually, at 2σ, the maximum value allowed for n is ~ 1.05 for $\Omega_{mo} = 0.35$. Hence, we may conclude that the original CW model ($n = 2$) is ruled out by the WMAP data.

References

[1] S. Perlmutter *et al.*, *ApJ* **517**, 565 (1999); A. Riess *et al.*, *Astron. J.* **116**, 1009 (1998).

[2] T. Padmanabhan, *Phys. Rep.* **380**, 235 (2003); P. J. E. Peebles and B. Ratra, *Rev. Mod. Phys.* **75**, 559 (2003); J. A. S. Lima, *Braz. J. Phys.* **34**, 194 (2004).

[3] S. Weinberg, *Rev. Mod. Phys.* **61**, 1 (1989); V. Sahni and A. A. Starobinsky, *IJMPD* **9**, 373 (2000).

[4] M. Ozer and O. Taha, *Nucl. Phys. B* **287**, 776 (1987); K. Freese *et al.*, *Nucl. Phys. B* **287**, 797 (1987); J. C. Carvalho, J. A. S. Lima and I. Waga, *Phys. Rev. D* **46**, 2404 (1992); J. A. S. Lima and J. M. F. Maia, *Phys. Rev. D* **49**, 5597 (1994); J. A. S. Lima and M. Trodden, *Phys. Rev. D* **53**, 4280 (1996); J. A. S. Lima, *Phys. Rev. D* **54**, 2571 (1996); J. M. Overduin and F. I. Cooperstock, *Phys. Rev. D* **58**, 043506 (1998); J. V. Cunha *et al.*, *A&A* **390**, 809 (2002); J. V. Cunha, J. A. S. Lima and J. S. Alcaniz, *Phys. Rev. D* **66**, 023520 (2002); J. V. Cunha and R. C. Santos, *Int. J. Mod. Phys. D* **13**, 1321 (2004); C. S. Camara *et al.*, *Phys. Rev. D* **69**, 123504 (2004); O. Bertolami, *Nuovo Cim. B* **93**, 36 (1986).

[5] J. S. Alcaniz and J. A. S. Lima, *ApJ* **521**, L87 (1999); J. S. Alcaniz and J. A. S. Lima, *ApJ* **550**, L133 (2001); J. A. S. Lima and J. S. Alcaniz, *MNRAS* **317**, 893 (2000); G. Sethi, A. Dev and D. Jain, *Phys. Lett. B* **624**, 135 (2005); D. Jain and A. Dev, *Phys. Lett. B* **633**, 436 (2006).

[6] Wei Chen and Yong-Shi Wu, *Phys. Rev. D* **41**, 695 (1990).

[7] P. Wang and Xin-He Meng, *Class. Quantum Grav.* **22**, 283 (2005).

[8] G. Hasinger, N. Schartel and S. Komossa, *ApJ* **573**, L77 (2002).

[9] A. Friaça, J. S. Alcaniz and J. A. S. Lima, *MNRAS* **362**, 1295 (2005).

[10] R. F. Sisteró, *GRG* **23**, 1265 (1991); J. A. S. Lima and J. M. F. Maia, *Mod. Phys. Lett. A* **8**, 591 (1993); J. Matyjasek, *Phys. Rev. D* **51**, 4154 (1995).

[11] C. L. Bennett *et al.*, *ApJ. Suppl.* **148**, 175 (2003).

[12] W. L. Freedman *et al.*, *ApJ* **553**, 47 (2001).

[13] J. S. Alcaniz, J. A. S. Lima and J. V. Cunha, *MNRAS* **340**, L39 (2003).

[14] J. S. Alcaniz and J. A. S. Lima, *Phys. Rev. D* **72**, 063516 (2005).